CBSE Class 12 Maths Question Paper 2020 Set 3

General Instructions:

Read the following instructions very carefully and strictly follow them:

(i) This question paper comprises **four** Sections A, B, C and D. This question paper carries **36** questions. **All** questions are compulsory.

(ii) **Section A –** Questions no. **1 to 20** comprises of **20** questions of **1** mark each.

(iii) **Section B –** Questions no. **21 to 26** comprises of **6** questions of **2** mark each.

(iv) **Section C –** Questions no. **27 to 32** comprises of **6** questions of **4** mark each.

(v) **Section D –** Questions no. **33 to 36** comprises of **4** questions of **6** mark each.

(vi) There is no overall choice in the question paper. However, an internal choice has been provided in 3 questions of one mark, 2 questions of two marks, 2 questions of four marks and 2 questions of six marks. Only one of the choices in such questions have to be attempted.

(vii) In addition to this, separate instructions are given with each section and question, wherever necessary.

(viii) Use of calculators is **not** permitted.

SECTION – A

Question number 1 to 20 carry 1 mark each.

Question number 1 to 10 are multiple choice type questions. Select the correct option.

1. The value of p for which $p\left(\hat{i}+\hat{j}+\hat{k}\right)$ is a unit vector is

(a) 0 (b) $\frac{1}{\sqrt{3}}$ (c) 1 (d) $\sqrt{3}$

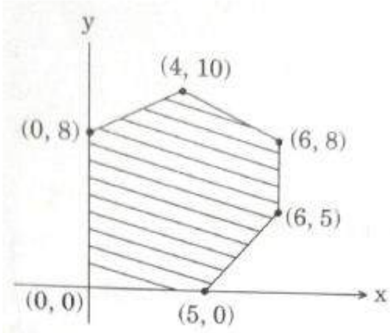
2. $\tan\left(\sin^{-1}\frac{3}{5}+\tan^{-1}\frac{3}{4}\right)$ is equal to

(a) $\frac{7}{24}$ (b) $\frac{24}{7}$ (c) $\frac{3}{2}$ (d) $\frac{3}{4}$

3. The feasible region for an LPP is shown below:

Let $z = 3x - 4y$ be the objective function. Minimum of z occurs at

DATE:	**SUBJECT**
CLASS:	
CENTRE:	**TOPIC**



(a) $(0,0)$ (b) $(0,8)$ (c) $(5,0)$ (d) $(4,10)$

4. If f and g are two functions from R to R defined as $f(x)=|x|+x$ and $g(x)=|x|-x,$ then $fog(x)$ for $x<0$ is

(a) $4x$ (b) $2x$ (c) 0 (d) $-4x$

5. $\int \frac{1}{x\log x}dx$ is equal to

(a) $\frac{(\log x)^2}{2}+c$ (b) $\log|\log x|+c$ (c) $\log|x\log x|+c$ (d) $\frac{1}{\log x}+c$

6. The order of the differential equation of the family of circles touching $x-$ axis at the origin is

(a) 1 (b) 2 (c) 3 (d) 4

7. If $A=\begin{bmatrix}-2 & 0 & 0\\ 0 & -2 & 0\\ 0 & 0 & -2\end{bmatrix}$, then the value of $|adj\ A|$ is

(a) 64 (b) 16 (c) 0 (d) -8

8. The image of the point $(2,-1,4)$ in the YZ-plane is

(a) $(0,-1,4)$ (b) $(-2,-1,4)$ (c) $(2,1,-4)$ (d) $(2,0,4)$

9. The maximum value of slope of the curve $y=-x^3+3x^2+12x-5$ is

(a) 15 (b) 12 (c) 9 (d) 0

10. The vector equation of XY-plane is

(a) $\vec{r}.\hat{k}=0$ (b) $\vec{r}.\hat{j}=0$ (c) $\vec{r}.\hat{i}=0$ (d) $\vec{r}.\vec{n}=1$

Fill in the blanks in question number 11 to 15.

11. The area of the parallelogram whose diagonals are $2\hat{i}$ and $-3\hat{k}$ is __________ square units.

(OR)

The value of λ for which the vectors $2\hat{i} - \lambda\hat{j} + \hat{k}$ and $i + 2\hat{j} - \hat{k}$ are orthogonal is __________.

12. A bag contains 3 black, 4 red and 2 green balls. If three balls are drawn simultaneously at random, then the probability that the balls are of different colours is __________.

13. The minimum value of the function $f(x) = |x+3| - 1$ is __________.

14. If $y = \tan^{-1} x + \cot^{-1} x,\ x \in R,$ then $\frac{dy}{dx}$ is equal to __________.

(OR)

If $\cos(xy) = k,$ where k is a constant and $xy \neq n\pi,\ n \in Z,$ then $\frac{dy}{dx}$ is equal to __________.

15. The value of λ sp that the function f defined by $f(x) = \begin{cases} \lambda x, & if \quad x \le \pi \\ \cos x, & if \quad x > \pi \end{cases}$ is continuous at $x = \pi$ is __________.

Question numbers 16 to 20 are very short answer type questions.

16. Evaluate: $\int_{-2}^{2} |x|\, dx$

(OR)

Find: $\int \frac{dx}{3 + 4x^2}$

17. Find the interval in which the function f given by $f(x) = 7 - 4x - x^2$ is strictly increasing.

18. Differentiate $\sin^2\left(\sqrt{x}\right)$ with respect to x.

19. Construct a 2×2 matrix $A = \left[a_{ij}\right]$ whose elements are given by $a_{ij} = \left|(i)^2 - j\right|$.

20. A black die and a red die are rolled together. Find the conditional probability of obtaining a sum greater than 9 given that the black die resulted in a 5.

SECTION – B

Question numbers 21 to 26 carry 2 marks each.

21. Show that for any two non-zero vectors $\vec{a}$ and $\vec{b}$, $\left|\vec{a} + \vec{b}\right| = \left|\vec{a} - \vec{b}\right|$ if $\vec{a}$ and $\vec{b}$ are perpendicular vectors.

(OR)

Show that the vectors $2\hat{i}-\hat{j}+\hat{k}$, $3\hat{i}+7\hat{j}+\hat{k}$ and $5\hat{i}+6\hat{j}+1\hat{k}$ form the sides of a right-angled triangle.

22. Find the matrix A such that $A\begin{bmatrix}1 & 2\\ -1 & 0\end{bmatrix}=\begin{bmatrix}3 & 4\\ -1 & 6\end{bmatrix}$.

23. If $y=\tan^{-1}\left[\frac{x}{\sqrt{a^2-x^2}}\right], |x|<a$, then find $\frac{dy}{dx}$.

24. If A and B are two events such that $P(A)=0.4, P(B)=0.3$ and $P(A\cup B)=0.6$, then find $P(B'\cap A)$.

25. Solve for x:

$$\sin^{-1}4x+\sin^{-1}3x=-\frac{\pi}{2}$$

(OR)

Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right), -\frac{3\pi}{2}<x<\frac{\pi}{2}$ in the simplest form.

26. Find the coordinates of the point where the line through (-1, 1, -8) and (5, -2, 10) crosses the ZX-plane.

SECTION - C

Question number 27 to 32 carry 4 marks each.

27. Solve the following LPP graphically:

Minimize $z=5x+7y$

subject to the constraints

$$2x+y\geq 8$$

$$x+2y\geq 10$$

$$x, y\geq 0$$

Solution:

28. Evaluate: $\int_{-1}^{3/2}|x\sin \pi x|\,dx$

29. A bag contains two coins, one biased and the other unbiased. When tossed, the biased coin has a 60% chance of showing heads. One of the coin is selected at random and on tossing it shows tails. What is the probability it was an unbiased coin?

(OR)

The probability distribution of a random variable X, where k is a constant is given below:

$$P(X=x)=\begin{cases} 0.1 & if & x=0 \\ kx^2, & if & x=1 \\ kx, & if & x=2\, or\, 3 \\ 0, & otherwise & \end{cases}$$

Determine

(a) the value of k

(b) $P(X \le 2)$

(c) Mean of the distribution

30. Find the particular solution of the differential equation $\frac{dy}{dx}+y\sec x=\tan x,$ where $x\in\left[0,\frac{\pi}{2}\right)$ given that $y=1,$ when $x=\frac{\pi}{4}.$

31. Show that the function $f:(-\infty,0)\to(-1,0)$ defined by $f(x)=\frac{x}{1+|x|}, x\in(-\infty,0)$ is one-one and onto.

(OR)

Show that the reaction R in the set $A=\{1,2,3,4,5,6\}$ given by $R=\{(a,b):|a-b|\, is\, divisible\, by\, 2\}$ is an equivalence relation.

32. If $y=x^3(\cos x)^x+\sin^{-1}\sqrt{x},$ find $\frac{dy}{dx}.$

SECTION – D

Question number 33 to 36 carry 6 marks each.

33. Find the points on the curve $9y^2=x^3,$ where the normal to the curve makes equal intercepts with both the axes. Also find the equation of the normals.

34. Show that the lines $\frac{x-2}{1}=\frac{y-2}{3}=\frac{z-3}{1}$ and $\frac{x-2}{1}=\frac{y-3}{4}=\frac{z-4}{2}$ intersect. Also, find the coordinates of the point of intersection. Find the equation of the plane containing the two lines.

35. Using integration, find the area of the parabola $y^2=4ax$ bounded by its latus rectum.

(OR)

Using integration, find the area of the region bounded by the curves $(x-1)^2+y^2=1$ and $x^2+y^2=1.$

36. Solve the following system of equations by matrix method:

$$x - y + 2z = 7$$

$$2x - y + 3z = 12$$

$$3x + 2y - z = 5$$

(OR)

Obtain the inverse of the following matrix using elementary operations:

$$A = \begin{bmatrix} 2 & 1 & -3 \\ -1 & -1 & 4 \\ 3 & 0 & 2 \end{bmatrix}$$

CBSE Class 12 Maths Question Paper 2020 Set 3 Solution

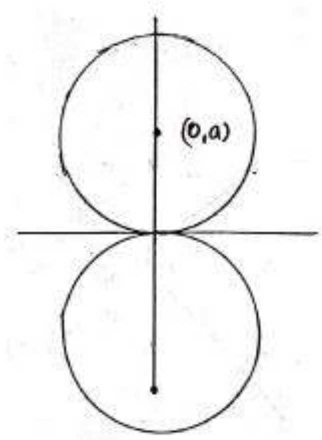
CLASS XII

MATHS SET – III : 65/3/3

S.NO	SOLUTION	MARK
1	**(B)** $p\sqrt{3}=1 \quad \Rightarrow \quad p=\frac{1}{\sqrt{3}}$	1
2	**(B)** $\tan\left(\sin^{-1}\frac{3}{5}+\tan^{-1}\frac{3}{4}\right)$ $=\tan\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{3}{4}\right)$ $=\tan\left(2\tan^{-1}\frac{3}{4}\right)$ $=\tan\left(\tan^{-1}\frac{2\left(3/4\right)}{1-\left(3/4\right)^2}\right)=\frac{\left(6/4\right)}{\frac{16-9}{16}}=\frac{24}{7}$	1
3	**(B)** $z=3x-4y$ at $(0,0)\Rightarrow z=0$ at $(0,8)\Rightarrow z=-32$ at $(5,0)\Rightarrow z=15$ at $(4,10)\Rightarrow z=-28$ Minimum $=-32$	1
4	**(D)** $f(x)=\lvert x\rvert+x=\begin{cases}2x & , x\ge 0\\ 0 & , x<0\end{cases}$ $g(x)=\lvert x\rvert-x=\begin{cases}0 & , x\ge 0\\ -2x & , x<0\end{cases}$ $f[g(x)]=\lvert x\rvert-x=\begin{cases}2:g(x) & , g(x)\ge 0\\ 0 & , g(x)<0\end{cases}$ $f[g(x)]=-4x \quad , \quad x<0$	1
5	**(B)** $\int\frac{1}{\log x}.dx$ Let $\log x=t \quad \Rightarrow \quad \frac{1}{x}.dx=dt$ $\int\frac{1}{\log x}.dx=\int\frac{dt}{t}=\log t+c=\log\lvert\log x\rvert+c$	1

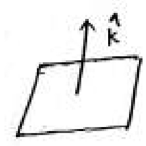
CLASS XII

MATHS SET – III : 65/3/3

6	**(A)** $x^2+(y-a)^2=a^2$ $x^2+y^2-2ay+a^2=a^2$ $x^2+y^2=2ay$ $2x+2y.\frac{dy}{dx}=2a.\frac{dy}{dx}$ $\frac{dy}{dx}=\frac{2x}{2a-2y}$ Order $=1$ 	1
7	**(A)** $A=\begin{bmatrix}-2&0&0\\0&-2&0\\0&0&-2\end{bmatrix}$ $\|A\|=-2(4-0)=-8$ $\|adj\ A\|=\|A\|^{3-1}=\|A\|^2=(-8)^2=64$	1
8	**(B)** Image of $(2,-1,4)$ in the YZ-plane is $(-2,-1,4)$	1
9	**(A)** $y=-x^3+3x^2+12x-5$ $\frac{dy}{dx}=-3x^2+6x+12$ $=-3(x^2-2x-4)$ $=-3((x-1)^2-5)$ $\frac{dy}{dx}=15-3(x-1)^2$	1

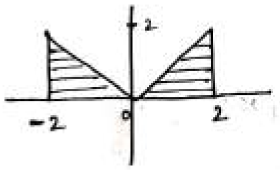
CLASS XII

MATHS SET – III : 65/3/3

	Maximum value $=15$	
10	**(A)** $\vec{r}.\hat{k}=0$ 	1
11	Area of parallelogram $=\frac{1}{2}\left\|d_1\times d_2\right\|=\frac{1}{2}\times 2\times 3=3$	1
	(OR) $\left(2\hat{i}-\lambda\hat{j}+\hat{k}\right).\left(\hat{i}+2\hat{j}-\hat{k}\right)=0\Rightarrow 2-2\lambda-1=0\Rightarrow\lambda=\frac{1}{2}$	1
12	$\frac{4c_1\times 3c_1\times 2c_1}{9c_3}=\frac{2}{7}$	1
13	$f(x)=\left\|x+3\right\|-1$ Minimum value $=-1$	1
14	$y=\tan^{-1}x+\cot^{-1}x$ $\frac{dy}{dx}=\frac{1}{1+x^2}-\frac{1}{1+x^2}=0$	1
	(OR) $y=\tan^{-1}x+\cot^{-1}x$ $y=\pi/2$ $\frac{dy}{dx}=0$	1
	(OR) $\cos(xy)=k \quad \Rightarrow -\sin(xy).\left(x\frac{dy}{dx}+y\right)=0$ $\Rightarrow -\sin(x.y).x\frac{dy}{dx}=y.\sin(xy)$ $\Rightarrow\frac{dy}{dx}=\frac{-y\sin(xy)}{x\sin(xy)}=\frac{-y}{x}$	1
15	$RHL=\cos\pi=-1$ $LHL=\lambda\pi$ $\Rightarrow\lambda\pi=-1 \quad \Rightarrow\lambda=-1/\pi$	1
16	$\int_{-2}^{2}\left\|x\right\|dx$	½

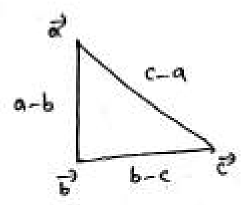
CLASS XII

MATHS SET – III : 65/3/3

	Area $=\left(\frac{1}{2}\times 2\times 2\right)+\left(\frac{1}{2}\times 2\times 2\right)$ $=4$ sq. units 	½
	(OR) $\int \frac{dx}{9+4x^2}=\frac{1}{4}\int \frac{dx}{9/4+x^2}=\frac{1}{4}.\frac{2}{3}\tan^{-1}\left(\frac{2x}{3}\right)$ $\int \frac{dx}{x^2+a^2}=\frac{1}{a}\tan^{-1}\left(x/a\right)+c=\frac{1}{6}\tan^{-1}\left(\frac{2x}{3}\right)$	½ ½
17	$f(x)=7-4x-x^2$ $f'(x)=-4-2x$ $f'(x)>0$ $-4-2x>0 \quad \Rightarrow -4>2x \quad \Rightarrow x<-2$	½ ½
18	$y=\sin^2\sqrt{x}$ $\frac{dy}{dx}=2\sin^2\sqrt{x}.\cos\sqrt{x}.\frac{1}{2\sqrt{x}}$ $\frac{dy}{dx}=\frac{\sin\sqrt{x}.\cos\sqrt{x}}{\sqrt{x}}$	1
19	$a_{ij}=\left\|(i)^2-j\right\|$ $a_{11}=1-1=0 \qquad a_{21}=4-1=3$ $a_{12}=\|1-2\|=1 \qquad a_{22}=4-2=2$ $\therefore \qquad A=\begin{bmatrix}0 & 1\\ 3 & 2\end{bmatrix}$	½ ½
20	Black die – 5 Red die – 5, 6 Probability $=\frac{2}{6}=\frac{1}{3}$	½ ½

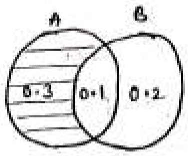
CLASS XII

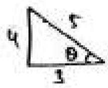
MATHS SET – III : 65/3/3

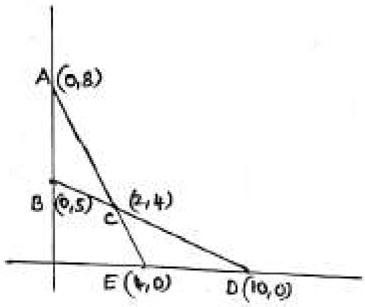
21	$\lvert a+b\rvert = \lvert a-b\rvert$	
	$a^2+b^2+2(ab) = a^2+b^2-2(ab)$	1
	$ab = 0$	
	$\therefore$ a and b are perpendicular	1
	(OR) $a-b = -\hat{i}-8\hat{j}$	
	$\lvert a-b\rvert \sqrt{1+64} = \sqrt{65}$	½
	$b-c = -2\hat{i}+\hat{j}-\hat{k}$	
	$\lvert b-c\rvert = \sqrt{4+1+4} = \sqrt{6}$	½
	$c-a = 3\hat{i}+7\hat{j}+\hat{k}$	
	$\lvert c-a\rvert = \sqrt{9+49+1} = \sqrt{59}$	
	$\lvert a-b\rvert^2 = \lvert b-a^1 + \lvert c-a\rvert^2$	
	$\therefore$ $\vec{a},\vec{b},\vec{c}$ are sides of Right angled $\Delta le.$	1
		
22	$A\begin{bmatrix}1 & 2\\ -1 & 0\end{bmatrix} = \begin{bmatrix}3 & 4\\ -1 & 6\end{bmatrix}$	
	Let $A = \begin{bmatrix}a & b\\ c & d\end{bmatrix}$	½
	$\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}1 & 2\\ -1 & 0\end{bmatrix} = \begin{bmatrix}3 & 4\\ -1 & 6\end{bmatrix}$	½
	$\begin{bmatrix}a-b & 2a\\ c-d & 2c\end{bmatrix} = \begin{bmatrix}3 & 4\\ -1 & 6\end{bmatrix}$	
	$a=2 \quad a-b=3 \quad \Rightarrow b=-1$	
	$c=3 \quad c-d=-1 \quad \Rightarrow d=4$	
	$\therefore \quad A = \begin{bmatrix}2 & -1\\ 3 & 4\end{bmatrix}$	1

CLASS XII

MATHS SET – III : 65/3/3

23	$y=\tan^{-1}\left[\dfrac{x}{\sqrt{a^2-x^2}}\right]$ Let $x=a\sin\theta \quad \Rightarrow \dfrac{x}{a}=\sin\theta \quad \Rightarrow \theta\sin^{-1}\left(x/a\right)$ $y=\tan^{-1}\left(\dfrac{a\sin\theta}{\sqrt{a^2-a^2\sin^2\theta}}\right)$ $y=\tan^{-1}(\tan\theta)$ $y=\theta \quad \Rightarrow y=\sin^{-1}\left(x/a\right)$ $\Rightarrow \dfrac{dy}{dx}=\dfrac{1}{\sqrt{1-x^2/a^2}}\cdot\dfrac{1}{a}=\dfrac{1}{\sqrt{a^2-x^2}}$	½ ½ 1
24	$P(A)=0.4$ $P(B)=0.3$ $P(A\cup B)=0.6$ $P(B'\cap A)=0.3$ 	1 1
25	$\sin^{-1}4x+\sin^{-1}(3x)=\dfrac{-\pi}{2}$ $\sin^{-1}4x+\dfrac{\pi}{2}-\cos^{-1}(3x)=\dfrac{-\pi}{2}$ $\sin^{-1}4x+\dfrac{-\pi}{2}-\dfrac{\pi}{2}+\cos^{-1}(3x)$ $\sin^{-1}(4x)+-\pi+\cos^{-1}(3x)$ $\sin^{-1}(4x)+-\left[\pi-\cos^{-1}3x\right]$ $\sin^{-1}(4x)+-\cos^{-1}(-3x)$ $\sin^{-1}(-4x)+\cos^{-1}(-3x)$ Let $\sin^{-1}(-4x)=\theta \qquad \cos^{-1}(-3x)=\theta$	½ ½ ½

	$-4x=\sin\theta \qquad -3x=\cos\theta$ $\dfrac{\sin\theta}{\cos\theta}=\dfrac{4}{3} \qquad \Rightarrow \tan\theta = 4/3 \qquad -4x = 4/5$ $x=\dfrac{-1}{5}$ 	½
	$\tan^{-1}\left(\dfrac{\cos x}{1-\sin x}\right)$ $=\tan^{-1}\left(\dfrac{\cos^2 x/2-\sin^2 x/2}{1-2\sin x/2 . \cos x/2}\right)$ $=\tan^{-1}\left(\dfrac{(\cos x/2+\sin x/2)(\cos x/2-\sin x/2)}{(\cos x/2-\sin x/2)^2}\right)$ $=\tan^{-1}\left(\dfrac{\cos x/2+\sin x/2}{\cos x/2-\sin x/2}\right)$ $=\tan^{-1}\left(\dfrac{1+\tan x/2}{1-\tan x/2}\right)$ $=\tan^{-1}\left[\tan\left(\pi/4+x/2\right)\right]$ $=\dfrac{\pi}{4}+\dfrac{x}{2}$	½ ½ ½ ½
26	On ZX plane y = 0 Dr's of the line $\to$ 6, -3, 18 Eqn of the line $\to \dfrac{x+1}{6}=\dfrac{y-1}{-3}=\dfrac{z+8}{18}=\lambda$ $x=6\lambda-1, y=-3\lambda+1, z=18\lambda-8$ $y=0 \quad \Rightarrow -3\lambda+1=0 \Rightarrow \lambda = 1/3$ $\therefore$ The point $=(1,0,-2)$	½ ½ 1
27	$2x+y=8\to(0,8),(4,0)$	1

	$2x+y>8 \rightarrow$ away from origin $x+2y=10 \rightarrow (0,5),(10,0)$ $x+2y>10 \rightarrow$ away from origin $z=5x+7y$ at $(0,8) \rightarrow z=56$ at $(2,4) \rightarrow z=38$ at $(10,0) \rightarrow z=50$ Minimum value = 38 at $c(2,4)$  	1 1 1
28	$\sin \pi x \begin{cases} <0 & -1<x<0 \\ >0 & 0<x<1 \\ <0 & 1<x<3/2 \end{cases}$ $x\sin \pi x \begin{cases} >0 & -1<x<0 \\ >0 & 0<x<1 \\ <0 & 1<x<3/2 \end{cases}$ $x\sin \pi x \begin{cases} >0 \ , & -1<x<1 \\ <0 \ , & 1<x<3/2 \end{cases}$ $I=2\int_0^1 x\sin \pi x.dx+\int_1^{3/2} -x\sin \pi y.dx$ $\int x\sin \pi x.dx=\frac{-x\cos \pi x}{\pi}+\int \frac{\cos \pi x}{\pi}.dx$ $=\frac{-x\cos \pi x}{\pi}+\int \frac{\sin \pi x}{\pi^2}$	1 1

CLASS XII

MATHS SET – III : 65/3/3

$$I = 2\left[\frac{-x\cos\pi x}{\pi} + \frac{\sin\pi x}{\pi^2}\right]_0^1 - \left[\frac{-x\cos\pi x}{\pi} + \frac{\sin\pi x}{\pi^2}\right]_1^{3/2}$$ 1

$$2\left[\left(\frac{-\cos\pi}{\pi} + \frac{\sin\pi}{\pi^2}\right) - (0+0)\right] - \left[\left(\frac{-3}{2\pi}.\cos\frac{3\pi}{2} + \frac{1}{\pi^2}\sin\frac{3\pi}{2}\right) - \left(\frac{-\cos\pi}{\pi} + \frac{\sin\pi}{\pi^2}\right)\right]$$

$$= \frac{-2}{\pi}\cos\pi - \frac{1}{\pi^2}\sin\frac{3\pi}{2} - \frac{\cos\pi}{\pi}$$

$$= \frac{-3}{\pi}(-1) - \frac{1}{\pi^2}(-1) = \frac{3}{\pi} + \frac{1}{\pi^2}$$ 1

29 (2)

	Head	Tail
Biased	0.6	0.4
Unbiased	0.5	0.5

$$P\left(\frac{U}{T}\right) = \frac{\frac{1}{2}\times 0.5}{\frac{1}{2}\times 0.4 + \frac{1}{2}\times 0.5} = \frac{1/4}{\frac{1}{5}+\frac{1}{4}} = \frac{1/4}{\frac{9}{20}} = \frac{1}{4}\times\frac{20}{9} = \frac{5}{9}$$ 2

30

$$\frac{dy}{dx} + y\sec x = \tan x$$

$$P = \sec x, \qquad Q = \tan x$$ 1

$$IF = e^{\int P.dx} = e^{\int \sec x.dx} = e^{\log|\sec x + \tan x|} = \sec x + \tan x$$

$$y.IF = \int Q.IF + C$$ 1

$$y(\sec x + \tan x) = \int \tan x(\sec x + \tan x) + C$$

$$y(\sec x + \tan x) = \int \tan x \sec x + \tan^2 x.dx + C$$

$$y(\sec x + \tan x) = \sec x + \int(\sec^2 x - 1)dx + C$$

$$y(\sec x + \tan x) = \sec x + \tan x - x + C$$ 1

at $y = 1$ and $x = \pi/4$

$$1(\sec\pi/4 + \tan\pi/4) = \sec\pi/4 + \tan\pi/4 - \pi/4 + C$$

$$\sqrt{2} + 1 = \sqrt{2} + 1 - \pi/4 + C \qquad \Rightarrow C = \pi/4$$

$$y(\sec x + \tan x) = \sec x + \tan x - x + \pi/4$$ 1

CLASS XII

MATHS SET – III : 65/3/3

31

$$f(x) = \frac{x}{1+|x|}$$

$$|x| = \begin{cases} x , & x \geq 0 \\ -x , & x < 0 \end{cases}$$

$$f(x) = \begin{cases} \dfrac{x}{1+x} , & x \geq 0 \\ \underline{x} , & x < 0 \end{cases}$$ 1

<u>one-one:</u>

For $x \geq 0$	For $x < 0$
$f(x_1) = f(x_2)$	$f(x_1) = f(x_2)$
$\frac{x_1}{1+x_1} = \frac{x_2}{1+x_2}$	$\frac{x_1}{1-x_1} = \frac{x_2}{1-x_2}$
$x_1 + x_1x_2 = x_2 + x_1x_2$	$x_1 - x_1x_2 = x_2 - x_1x_2$
$x_1 = x_2$	$x_1 = x_2$

1

Hence $f(x_1) = f(x_2) \Rightarrow x_1 x_2$

$\therefore$ f is one-one

<u>onto:</u>

For $x \geq 0$	For $x < 0$
Let $f(x) = y$	Let $f(x) = y$
$y = \frac{x}{1+x}$	$y = \frac{x}{1-x}$
$y + xy = x$	$y - xy = x$
$y = x(1-y)$	$y = x(1+y)$
$x = \frac{y}{1-y}$	$x = \frac{y}{1+y}$

1

$\therefore$ f is onto.

Hence f is both one-one and onto. 1

(OR)

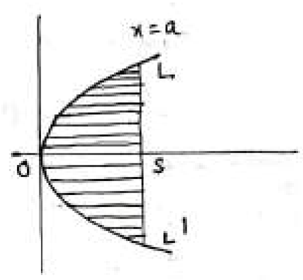
CLASS XII

MATHS SET – III : 65/3/3

32	$y = x^3(\cos x)^x + \sin^{-1}\sqrt{x}$ Let $u = (\cos x)^x \quad \Rightarrow \log u = x.\log(\cos x)$ $\Rightarrow \frac{1}{4}.\frac{du}{dx} = x\frac{1}{\cos x}(-\sin x) + \log(\cos x)$ $\Rightarrow \frac{du}{dx} = (\cos x)^x[\log(\cos x) - x\tan x]$ Now, $y = x^3(\cos x)^x + \sin^{-1}\sqrt{x}$ $\frac{dy}{dx} = x^3(\cos x)^x[\log(\cos x) - \tan x] + 3x^2(\cos x)^x + \frac{1}{\sqrt{1-x}}.\frac{1}{2\sqrt{x}}$	1 1 1 1
33	$9y^2 = x^3 \qquad \to (i)$ $18y.\frac{dy}{dx} = 3x^2$ Given $m = \pm 1$ $\frac{-6y}{x^2} = \pm 1$ $\frac{-6y}{x^2} = 1$ or $\frac{-6y}{x^2} = -1$ $x^2 = -6y$ or $x^2 = 6y$ Substitute the above in (i) $9\left(\frac{x^4}{36}\right) = x^3 \quad \Rightarrow \quad x = 0$ or 4 If $x = 4 \quad \Rightarrow y = \pm\frac{8}{3}$ Equation of normal $\Rightarrow y - y_1 = \frac{-dx}{dy}(x - x_1)$ $\Rightarrow y - 8/3 = \frac{-6\left(\frac{8}{3}\right)}{16}(x-4)$ $\Rightarrow \frac{3y-8}{3} = -x + 4$ $\Rightarrow 3y - 8 = -3x + 12$ $\Rightarrow 3x + 3y = 20$	1 1 1 1 1 1

CLASS XII

MATHS SET – III : 65/3/3

34	$\frac{x-2}{1}=\frac{y-2}{3}=\frac{2-3}{1}=\lambda$ and $\frac{x-2}{1}=\frac{y-3}{4}=\frac{z-4}{2}=\mu$ $x=\lambda+2 \qquad x=\mu+2$ $y=3\lambda+2 \qquad y=4\mu+3$ $z=\lambda+3 \qquad z=2\mu+4$ $\lambda+2=\mu+2 \Rightarrow \lambda=\mu$ $3\lambda+2=4\mu+3 \Rightarrow \lambda=\mu=-1$ $\lambda+3=2\mu+4 \Rightarrow 2=2$ $\therefore$ The lines are intersect at $(1,-1,2)$ Equation of plane is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_1 & m_1 & n_1 \\ x_2 & m_2 & n_2 \end{vmatrix}=0 \quad \Rightarrow \begin{vmatrix} x-2 & y-2 & z-3 \\ 1 & 3 & 1 \\ 1 & 4 & 2 \end{vmatrix}=0$ $\Rightarrow 2x-y+z=5$	1 1 1 1 1 1
35	For parabola $y^2=4ax$ Latus rectum is $x=a$ Area = Area OLSL' = 2 × Area OSL $=2\int_0^a y.dx$ $=4\sqrt{a}\int_0^a x^{1/2}.dx$ $=\frac{8}{3}\sqrt{a}\left[x^{3/2}\right]_0^a=\frac{8}{3}a^2$ 	1 1 2 1 1
	(OR)	
36	$x-y+2z=7$ $2x-y+3z=12$	

$3x+2y-z=5$

$$\begin{bmatrix}1 & -1 & 2\\ 2 & -1 & 3\\ 3 & 2 & -1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}7\\ 12\\ 5\end{bmatrix}$$ 1

$|A|=1(1-6)+1(-2-9)+2(4+3)$

$=-5-11+14=-2$ 1

$$adj\ A=\begin{bmatrix}-5 & 11 & 7\\ 3 & -7 & -5\\ -1 & 1 & 1\end{bmatrix}^{-1}=\begin{bmatrix}-5 & 3 & -1\\ 11 & -7 & 1\\ 7 & -5 & 1\end{bmatrix}$$ 1

$$A^{-1}=\frac{adj\ A}{|A|}=\frac{-1}{2}\begin{bmatrix}-5 & 3 & -1\\ 11 & -7 & 1\\ 7 & -5 & 1\end{bmatrix}$$ 1

$$x=A^{-1}.B=\frac{-1}{2}\begin{bmatrix}-5 & 3 & -1\\ 11 & -7 & 1\\ 7 & -5 & 1\end{bmatrix}\begin{bmatrix}7\\ 12\\ 5\end{bmatrix}$$ 1

$$=\frac{-1}{2}\begin{bmatrix}-35+36-5\\ 77-84+5\\ 49-60+5\end{bmatrix}=\begin{bmatrix}2\\ 1\\ 3\end{bmatrix}$$

$\therefore \quad x=2, y=1, 2=3.$ 1